Problem set #1 - Tutorial

This is the first problem of the AT2, it goes through the very basics of what you will have to learn during this class. We will create a script, with an array, a loop and some basic plotting. Try and get used to writing down answers documented by figures in any of your favorite word processor. This is the most important part of the project reports grading you will have to hand in.

Problem 1  A simple population growth model (Niveau 1)

We will try to simulate a population of animals growing. We start with a fixed number of animals \( N_0 = 2 \), and we will model the growth yearly over a hundred years period. First, create a script.

Now for the growth model, let’s name \( p_n \) the population at year \( n \). It is safe to assume that the population will grow by a factor of itself, that is, the bigger the population, the bigger the growth. So the number of new individuals born any given year should look something like \( \alpha p \), if \( p \) is the population that year. Let us set \( \alpha \) to 0.1 for now. Noting the current year \( n - 1 \), this means we try to model the following map:

\[
p_n = p_{n-1} + 0.1 p_{n-1}
\]

1. Create an array named \( p \), meant to hold on to the population values every year.
2. Set the first value of the array, \( p_0 \) to the initial population value.
3. Given the map above, set the second value of the array by computing \( p_1 \).

To do a longer simulation we will have to use a loop. A loop repeats the same chunk of code over and over. So we have some control over what happens, a variable (an integer) gets \textit{incremented} (it will count 0, 1, 2, ..., until the bound we set). This integer is usually called \textit{index}. To keep track of what we did in the loop, it is useful to create arrays, that can hold on to multiple values at the same time. Arrays have a fixed size, hence they must be \textit{initialized} first. For this we need to know their size prior to the simulation.

This simple chunk of code creates an array of size 10 and writes 0, 1, ..., 9 inside the array elements:

```python
import numpy as np # import the library
myarray = np.zeros(10) # create the array
for i in range(10): # write the loop
    myarray[i] = i # write the index of the loop in the array
print myarray
```

The above example should help you through the following questions. Notice how the array has length 10, but the loop indices only reach 9, this is because the first element of arrays (in Python, not in MATLAB) is “myarray[0]”, not “myarray[1]".
1. Write down a loop that automates the computation of $p_1$ that you did before, to compute the remaining $p_2 \ldots p_{100}$ values.

2. Plot the result and show it.

3. What is happening? Could we have predicted it?

4. Play around by changing the growth parameter $\alpha$, how does it affect the result? Does changing the initial population affect the result sensibly?

Of course our model is very naive, and it seems that we could refine it a little bit. Let’s consider the general problem of resources, our population now lives in an environment where resources are limited. A good way of modeling that is simply to modulate the growth factor. This means that we will replace our $\alpha$ by some function of the population. We choose to write $\alpha = 200 - p$.

1. Plot this function for population rates between 0 and 500 individuals.

2. Interpret the curve, what does it imply for the growth rate of our population?

Now we have some intuition of the behavior of the population, but we might want to check it by simulating it again. Every next year the population grows by $\delta_n = 0.001p_{n-1}(200 - p_{n-1})$. This means the population follows the map.

$$p_n = p_{n-1} + 0.001p_{n-1}(200 - p_{n-1})$$

1. Simulate the system, and plot the variation of the population values over time. What happens?

2. Play around with the additional parameter that we have introduced, how does it affect the simulation result?

3. Play around with the initial number of individual, can it also change the general trend of the system?