CO6: Introduction to Computational Neuroscience

http://iec-lnc.ens.fr/group-for-neural-theory/teaching-260

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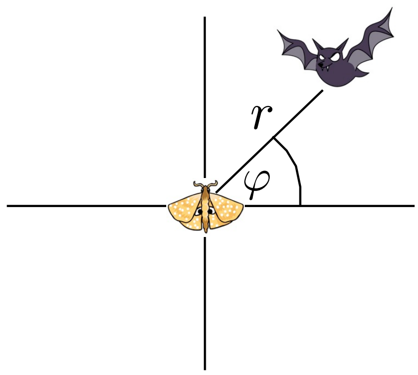
Exercise Sheet 7 — 9 May 2017

Please submit your solution in the next class (16 May 2016)

1. **The Nightly War of Moths and Bats.** Moths, like any other animal, do not like to be eaten. Unfortunately for many of them, they are being hunted by bats at night. Bats use ultrasound echolocation to detect moths in flight and catch them. Over the course of evolution, many moth species have developed very simple ears that essentially act as bat detectors—they have one or two auditory receptor neurons that are only sensitive to high-frequency sounds. The moth's response to an incoming bat is simple: if the bat is far, it will simply fly away from the sound. If the bat is relatively close, the moth will fold its wings and let itself drop onto the ground (see for instance: Roeder KD, 1963, *Nerve Cells and Insect Behavior*, Harvard University Press).

   We want to think about how the moth retrieves the relevant information (distance and incoming angle of the bat) from the sound intensity it picks up on its left and right ear. Given a bat coming from angle φ at distance r, we assume that the sound intensities impinging on the moth's two ears are given by:



$$I_R = \frac{I}{r}\left(\cos^2(\varphi/2) + \alpha\right)$$
$$I_L = \frac{I}{r}\left(\sin^2(\varphi/2) + \alpha\right)$$

   We ignore the problem of how the moth distinguishes between sounds coming from the front or the back and will simply focus on the front ($\varphi \in [0, 180]$).

   (a) Sketch the intensities on the left and right ear against the angle of the incoming bat for a fixed distance.

   (b) Sketch the intensities I_L and I_R against the distance of the incoming bat for a fixed angle.

   (c) What computations does the moth have to perform on I_L and I_R to extract the angle φ of the incoming bat and its distance r? What does the neuron compute that decides whether the moth should let itself fall to the ground?

2. **Parameter estimation.** In preparation of the next lectures, we will investigate how to fit the parameters of a simple neural input-output model. We will start with the simplest possible case, a single neuron with firing rate r that receives a single input s and fires according to the rule

$$r = ws$$

where w is an unknown "weight" parameter. In each trial, we present a stimulus s_i, where i denotes the number of the trial, and we record a response r_i. Given such a set of stimuli and responses (say there were M trials), we want to estimate the weight w so that the mean-square error

$$E(w) = \sum_{i=1}^{M} (r_i - ws_i)^2$$

is minimized. We will call this value of w the *optimal* w. Note that E is just a function of w, because there is only one variable (parameter). This problem is the simplest version of the *linear regression* problem.

(a) Compute the derivative $\mathrm{d}E(w)/\mathrm{d}w$.

(b) Sketch $E(w)$ as a function of w and sketch $\mathrm{d}E(w)/\mathrm{d}w$ as a function of w (just sketch the general shapes!).

(c) In many scientific labs you will find some person who knows everything better than you. Let's call this guy (it's usually a him) "Smarty Pants". Smarty Pants claims that the correct value is w_{sp}. Given your measurements, but *without* yet knowing the optimal value w_{opt}, how can you tell him whether the optimal value is smaller, larger or equal to Smarty Pants' guess? (Hint: think of (a))

(d) The above intuition allows an "iterative" solution to the problem of finding the optimal w. Assume you were given an initial guess $w = w_0$, write down an "update" equation that will change w by a small amount and thereby decrease $E(w)$. If you repeat ("iterate") this update equation, what is going to happen? (What does this remind you of?)

(e) The iterative solution from (d) is the method of choice for many optimization problems. The linear regression problem, however, is one of the few problems that can be solved analytically. Write down the exact solution for the optimal $w = w_{\mathrm{opt}}$, i.e., the paramater w that minimizes $E(w)$.