3.1. Biophysics of neurons
What you will see later: the binary neuron

\[ I = \sum_{k=1}^{n} w_k x_k \]

\[ y = H(\sum_{k=1}^{n} w_k x_k + b) \]
Electrical activity in vivo

Regular Spiking Pyramidal Cell: Visual Response
The action potential

Hodgkin & Huxley (1939)
Giant squid axon

Resting potential: the neuron is polarized
Refractory period

Membrane potential
\[ V_m = V_{in} - V_{out} \]
Electrical activity in vitro

Regular Spiking Pyramidal Cell: Current Pulses

Membrane potential

Injected current
How does a neuron process spike trains?

Input = \( N \) spike trains

Output = 1 spike train
How does a neuron process spike trains?

- Action potential (AP) or «spike»
- Postsynaptic potential (PSP)
- Firing threshold
- Temporal integration
Neuronal electricity
The resting potential

- At rest, the neuron is *polarized*: $V_m \approx -70 \text{ mV}$

**Terminology:**
- « depolarized »: $V_m$ increases
- « hyperpolarized »: $V_m$ decreases
Where does electricity come from?

- The membrane: lipid bilayer (insulator) with pores (channels = proteins)

Different ion concentrations inside and outside the cell (maintained by « ionic pumps »)

Ions carry electrical charges
Electrodiffusion through a semi-permeable membrane

(A- = anion)
Electrodiffusion through a semi-permeable membrane

1) Thermal agitation: K+ diffuses towards less concentrated side

2) Electrical field develops because of difference in charge

3) Electrical field opposes further diffusion: equilibrium

(A- = anion)
The «Nernst potential»

Equilibrium potential or «Nernst potential»:

or reversal potential

\[ E = \frac{RT}{zF} \ln \frac{C_{out}}{C_{in}} \]

\( F = 96\,000\, \text{C.mol}^{-1} \) (Faraday constant)
\( R = 8.314\, \text{J.K}^{-1}.\text{mol}^{-1} \) (gas constant)
\( z = \) charge of ion
Equivalent electrical circuit

Linear approximation of leak current: \( I = g_L(V_m - E_L) \)

\( E_L \approx -70 \text{ mV} \): the membrane is « polarized » \( (V_{in} < V_{out}) \)
Physics reminder

**Ohm’s Law:**

The current flowing through a resistor is directly proportional to the voltage drop across the resistor.

\[ I = \frac{V}{R} \quad \quad R = \frac{1}{g} \]

**Kirchhoff’s Law:**

The sum of currents flowing into a point is equal to the sum of currents flowing out of that point.

\[ I_1 + I_2 + I_3 + \cdots = 0 \]
The membrane equation

\[ C \frac{dV_m}{dt} + \frac{(V_m - E_L)}{R} = I_{inj} \]

\[ \tau \frac{dV_m}{dt} = E_L - V_m + RI_{inj} \]

\[ \tau = RC \]

membrane time constant (typically 3-100 ms)
Response to a current pulse

\[ \tau \frac{dV_m}{dt} = E_L - V_m + RI_{inj} \]

\[ V_\infty = E_L + RI \]

\[ V_m(t) = V_\infty + (V_m(0) - V_\infty)e^{-t/\tau} \]
Spikes

The membrane equation:

$$\tau \frac{dV_m}{dt} = E_L - V_m + RI$$

Injection of a current pulse:

```
-60 mV
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Threshold $V_t$

Reset $V_r$

« Integrate-and-fire » model:

If $V = V_t$ (threshold)
then: neuron spikes and $V \rightarrow V_r$ (reset)

(phenomenological description of action potentials)
Synapses
Synapses

Axosomatic synapses
Axodendritic synapses
A xo- axonic synapse

presynaptic bouton
transmitter
receptor
postsynaptic membrane
Synaptic currents

Transfer of charge (=ions) : \( Q = C \cdot \Delta V \)

\[
\tau \frac{dV_m}{dt} = E_L - V_m
\]

\( V_m \rightarrow V_m + \frac{RQ}{\tau} \) at spike time

\[
\Delta V = \frac{Q}{C} = \frac{RQ}{\tau}
\]

\[
V_m(t) = E_L + \frac{RQ}{\tau} e^{-\frac{t}{\tau}}
\]
Excitation/inhibition

\[ V_m(t) = E_L + \frac{RQ}{\tau} e^{-\frac{t}{\tau}} \]

Positive ions enter (Na+): excitatory (or negative ions exit)

Positive ions exit (K+): inhibitory (or negative ions enter)

In fact, it depends on the equilibrium potential of the synaptic channel.
### Major synapse types

<table>
<thead>
<tr>
<th>Neurotransmitter</th>
<th>Receptor</th>
<th>$E_{rev}$</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glutamate</td>
<td>AMPA</td>
<td>0 mV</td>
<td>fast</td>
</tr>
<tr>
<td></td>
<td>NMDA</td>
<td>0 mV</td>
<td>slow, voltage-dependent</td>
</tr>
<tr>
<td>GABA</td>
<td>GABA-A</td>
<td>-70 mV</td>
<td>fast</td>
</tr>
<tr>
<td></td>
<td>GABA-B</td>
<td>-100 mV</td>
<td>slow</td>
</tr>
</tbody>
</table>

Also: glycine, acetylcholine…

can change because it depends on ionic concentrations

Dale’s principle: a neuron only expresses one type of neurotransmitter
Several spikes

\[ \tau \frac{dV_m}{dt} = E_L - V_m \]
\[ V_m \rightarrow V_m + w \] at spike time
\[ w = \text{« synaptic weight »} \]

Many spikes at times \( t_i^k \) (i = synapse; k = spike index):
\[ V_m \rightarrow V_m + w_i \] at time \( t_i^k \)
\[ V_m(t) = E_L + \sum_{i,k} PSP_i(t - t_i^k) \]

Where:
\[ PSP_i(t) = H(t)w_i e^{-\frac{t}{\tau}} \]
\[ (H = \text{Heavyside}) \]
Coincidence detection
Coincidence detection: principle

\[ \tau \frac{dV}{dt} = E_L - V \]

\[ V \rightarrow V + w \]

Threshold \( V_t \)

Spike if \( |d| < -\tau \log \left( \frac{V_t - E_L}{w} - 1 \right) \)

\( \left( \frac{V_t - E_L}{2} < w < V_t - E_L \right) \)
Example in sound localization

Interaural time difference (ITD) depends on sound direction

sound

spikes
The Jeffress model of sound localization

When the sum of acoustic and neural transmission delays of one side equals that of the other side, the coincidence detector fires maximally.
Jeffress model in the cat

Jeffress (1948)
Coincidence model

Medial superior olive
Integration
Temporal integration

Many input spikes within the time constant: inputs are summed.

\[ \tau \frac{dV_m}{dt} = E_L - V_m \]

\[ V_m \rightarrow V_m + w \] at spike time

Sum dominates at short timescale
The firing rate

\[ T = \text{interspike interval (ISI)} \]

Firing rate

\[ F = 10 \text{ spikes/ 100 ms} = 100 \text{ Hz} \]

\[ F = \frac{1}{\langle T_n \rangle} \]

\( (T_n = t_{n+1}-t_n) \)
Firing rate for constant current

\[ \tau \frac{dV_m}{dt} = E_L - V_m + RI \]

threshold \( V_t \)
reset \( V_r \)

- **Firing condition:**
  \[ E_L + RI = V_t \]
  \[ I = \frac{V_t - E_L}{R} \]  
  « Rheobase current »

- **Time to threshold:**
  \[ T = \tau \log \frac{E_L + RI - V_m(0)}{E_L + RI - V_t} \]
  \[ T = \tau \log \frac{E_L + RI - V_r}{E_L + RI - V_t} \] from reset
Current-frequency relationship

\[ F = \frac{1}{T} = \left( \tau \log \frac{E_L + RI - V_r}{E_L + RI - V_t} \right)^{-1} \]
The refractory period

For a few ms after a spike: the neuron cannot produce a spike.
Firing rate with a refractory period

\[ T = \Delta + \tau \log \frac{E_L + RI - V_r}{E_L + RI - V_t} \text{ from reset} \]

\[ F = \frac{1}{T} = \left( \Delta + \tau \log \frac{E_L + RI - V_r}{E_L + RI - V_t} \right)^{-1} \quad \max 1/\Delta \]
Rank order coding
The neural « code »

Code:
- spike count (rate code)
- spike timing (temporal code)
- spike rank (rank order code)

Decoding rank order

- How to distinguish between AB and BA?

- Solution: excitation and inhibition
Prey localization by the sand scorpion

Inhibition of opposite neuron → more spikes near the source

(polar representation of firing rates)
Conversion rank order code → rate code

Stürtzl et al. (2000). Theory of arachnid prey localization. PRL
Neurons are sensitive to coincidences at the timescale of their membrane time constant.

Within the membrane time constant, inputs are summed.

Neurons are sensitive to the order of excitatory and inhibitory inputs.
Learning
Hebb’s rule

When an axon of cell A is near enough to excite cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A’s efficiency, as one of the cells firing B, is increased. (1949)

Neuron A and neuron B are active: $w_{AB}$ increases

Physiologically: « synaptic plasticity »

PSP size is increased
(or: transmission probability is increased)
Synaptic plasticity at spike level

- causal rule
- favors synchronous inputs

Presynaptic action potential

pre → post: potentiation
post → pre: depression

(STDP = Spike-Timing-Dependent Plasticity)

Dan & Poo (2006)
The precision of spike timing

Mainen & Sejnowski (1995)
(cortical neuron in vitro, somatic injection)

The same variable current is injected 25 times.

Spike timing is reproducible even after 1 s.

IF model: