Exercise Sheet 3 — 28 March 2017

Please submit your solution in the next class (4 April 2017)

1. Perceptrons

   (a) Consider the following two sets of two-dimensional patterns:

   \[ D_{\text{blue}} = \{(1, 2), (1, 3), (3, 4), (2, 4)\} \]
   \[ D_{\text{red}} = \{(4, 0), (6, 2), (2, -3), (6, -2)\} \]

   Are the two sets linearly separable? If they are, use a perceptron rule to find a readout weight vector that allows a binary neuron to classify the two sets in two different categories.

   (b) Repeat the exercise with the following two sets of three-dimensional patterns:

   \[ D_{\text{blue}} = \{(2, 1, 2), (3, 2, 3), (4, 2, 4), (3, 1, 2)\} \]
   \[ D_{\text{red}} = \{(1, 4, -1), (2, 3, 0), (1, 2, 0), (0, 1, 0)\} \]

   (c) Optional: Code the perceptron rule in a computer, and use the datasets of exercises (a) and (b) to check the validity of your answers. Please attach your code and the results obtained.

2. Hopfield networks

   (a) Consider a recurrent network of five binary neurons. Use the Hopfield rule to determine the synaptic weights of the network so that the pattern \( \xi^* = (1, -1, -1, 1, -1) \) is memorized. Show explicitly that \( \xi^* \) is a fixed point of the dynamics.

   (b) What is the output of the network if the dynamics start from the initial state \( \xi = (1, -1, -1, 1, -1) \)?

   (c) Using the Hopfield rule, modify the synaptic weights so that \( \eta^* = (1, 1, -1, -1, -1) \) is also memorized. Verify that both \( \xi^* \) and \( \eta^* \) are fixed points of the dynamics.

   (d) What is the output of the network if the dynamics start from \( \xi = (1, -1, 1, 1, 1) \)?