CO6: Introduction to Computational Neuroscience

http://iec-lnc.ens.fr/group-for-neural-theory/teaching-260

Lecturer & tutor: Grégory Dumont
gregory.dumont@ens.fr
Ecole Normale Supérieure
29 rue d'Ulm, 2nd floor

Exercise Sheet 3 — 28 March 2017

Please submit your solution in the next class (4 April 2017)

1. **Perceptrons**

   (a) Consider the following two sets of two-dimensional patterns:

   $$\mathcal{D}_{\text{blue}} = \{(1,2),(1,3),(3,4),(2,4)\},$$
   $$\mathcal{D}_{\text{red}} = \{(4,0),(6,2),(2,-3),(6,-2)\}.$$

   Are the two sets linearly separable? If they are, use a perceptron rule to find a readout weight vector that allows a binary neuron to classify the two sets in two different categories.

   (b) Repeat the exercise with the following two sets of *three*-dimensional patterns:

   $$\mathcal{D}_{\text{blue}} = \{(2,1,2),(3,2,3),(4,2,4),(3,1,2)\},$$
   $$\mathcal{D}_{\text{red}} = \{(1,4,-1),(2,3,0),(1,2,0),(0,1,0)\}.$$

   (c) **Optional**: Code the perceptron rule in a computer, and use the datasets of exercises (a) and (b) to check the validity of your answers. Please attach your code and the results obtained.

2. **Hopfield networks**

   (a) Consider a recurrent network of five binary neurons. Use the Hopfield rule to determine the synaptic weights of the network so that the pattern $\boldsymbol{\xi}^* = (1,-1,-1,1,-1)$ is memorized. Show explicitly that $\boldsymbol{\xi}^*$ is a fixed point of the dynamics.

   (b) What is the output of the network if the dynamics start from the initial state $\boldsymbol{\xi} = (1,-1,-1,1,-1)$?

   (c) Using the Hopfield rule, modify the synaptic weights so that $\boldsymbol{\eta}^* = (1,1,-1,-1,-1)$ is also memorized. Verify that both $\boldsymbol{\xi}^*$ and $\boldsymbol{\eta}^*$ are fixed points of the dynamics.

   (d) What is the output of the network if the dynamics start from $\boldsymbol{\xi} = (1,-1,1,1,1)$?