Exercise Sheet 7 — 10 May 2016

Please submit your solution in the next class (17 May 2016)

1. **Equilibrium potential.** Assume a neuron has both excitatory and inhibitory synaptic inputs, described by the conductances $g_{\text{exc}}$ and $g_{\text{inh}}$. Assume that the excitatory neurotransmitters open Na$^+$ channels with a reversal potential of $E_{\text{exc}} = 55$ mV, and the inhibitory neurotransmitters open K$^+$ channels with a reversal potential of $E_{\text{inh}} = -75$ mV. Plot the neuron’s equilibrium potential as a function of $g_{\text{exc}}$. (Hint: set $g_{\text{inh}} = 1\text{mS}$, then let $g_{\text{exc}}$ run from 0 to infinity). What happens for very small $g_{\text{exc}}$, what happens for large $g_{\text{exc}}$?

2. **Integrate-and-fire neuron.** We consider the integrate-and-fire neuron with differential equation

$$\tau \frac{dV(t)}{dt} = E_L - V(t) + RI(t) \quad (1)$$

Whenever the voltage reaches a threshold, $V \geq V_{\text{th}}$, it is set back to the equilibrium potential, $V = E_L$.

(a) Consider a constant current $I(t) = I_0$. How large must $I_0$ be so that the neuron starts spiking?

(b) Let us now consider the case of a current step. In this case, we turn on a current at time $t = t_0$, and then turn it off after $T$ time units. In other words, the current is given by

$$I(t) = \begin{cases} 
0 & \text{for } t < t_0 \\
I_0 & \text{for } t_0 \leq t < t_0 + T \\
0 & \text{for } t \geq t_0 + T 
\end{cases} \quad (2)$$

How large does $I_0$ now have to be, as a function of $T$, for the neuron to spike? Make a plot of $I_0$ versus $T$.

(c) **Advanced.** Imagine that two current steps of length $T$ are injected with a time delay of $\Delta t$. What is the maximum delay $\Delta t$ between the two steps until an action potential is emitted?

3. **Integrate-and-fire with refractory period.** Real neurons usually have refractory periods, i.e., for a few milliseconds after an action potential, the neuron will not fire again. How could you add such a refractory period to the integrate-and-fire neuron? Compute the new fI-curve, i.e., the firing rate versus input current curve of the neuron with refractory period. How does the new fI-curve differ from the old one?