(1) **Integrate-and-Fire Neuron** The leaky integrate-and-fire (LIF) neuron is probably one of the most famous spiking neuron models. In its simplest form, a neuron is modeled as a “leaky integrator” of its input $I(t)$

$$
\tau_m \frac{d}{dt} v(t) = E_L - v(t) + RI(t),
$$

(1)

where $v(t)$ represents the membrane potential at time $t$, $\tau_m$ is the membrane time constant, $R$ is the membrane resistance and $E_L$ the reversal potential. The spiking events are not explicitly modeled in the LIF model. Instead, when the membrane potential $v(t)$ reaches a certain threshold $v_{th}$ (spiking threshold), it is instantaneously reset to a lower value $v_r$ (reset potential) and the leaky integration process described by Equation 1 starts anew with the initial value $v_r$.

- Consider the case of constant input: $I(t) = I$, show that the solution of Equation 1 is then given by:

$$
v(t) = v_\infty + (v(0) - v_\infty) e^{-t/\tau_m}, \quad \text{with} \quad v_\infty = E_L + RI.
$$

(2)

- Deduce from Equation 2 the firing condition of the LIF neuron.
- Assuming that the initial voltage value is $v(0) = v_r$, establish the amount of time necessary for the LIF neuron to produce a spike (time to threshold).
- Deduce the firing rate of the LIF neuron.

(2) **Integrate-and-Fire with refractory period**: Real neurons usually have refractory periods, i.e., for a few milliseconds after an action potential, the neuron will not fire again. How could you add such a refractory period to the integrate-and-fire neuron? Compute the new fI-curve, i.e., the firing rate versus input current curve of the neuron with refractory period. How does the new fI-curve differ from the old one?